Module III

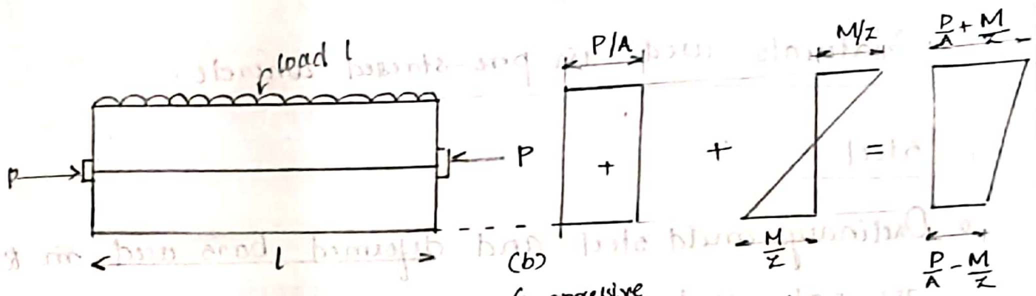
PRE STRESSED CONCRETE

- In ordinary reinforced ~~con~~ cement concrete, compression stresses are taken up by concrete and tensile stresses by steel alone. ~~The concr~~
- The concrete below the neutral axis is ignored since it is weak in tension.
- Although steel takes up the tensile stresses, the concrete in the tensile zone develops minute cracks.
- The load carrying capacity of such concrete sections can be increased if steel and concrete both are stressed before the application of external loads.
- This is the concept of prestressed concrete.
- As per ACI committee, pre stressed concrete is that concrete in which internal stresses of suitable magnitude are introduced so that the stresses resulting from the external loadings can be counteracted to a desired degree.

- In RCC members, pre-stressed induced is in compressive nature so that it balances the tensile stresses produced due to external load.
- It makes the whole section effective in resisting loads.

Concept of Pre stressing

- In reinforced concrete, pre stressed is commonly introduced by tension in reinforcement.
- So compression is induced in the zones where external loads would normally cause tensile stresses.



(a) Prestressed concrete beam (b) Compressive stress (c) Bending stress due to load (d) Final stress distribution

Compressive pre-stress in the beam $= +\frac{P}{A}$

Bending stress due to external loads $= \pm\frac{M}{I}\times y$ or $\frac{M}{Z}$

Net stress $= \frac{P}{A} + \frac{M}{I}y$

Net stress at the top fibre of the beam section $= \frac{P}{A} + \frac{M}{Z}$

$= \frac{P}{A} + \frac{M}{I}y$

Net stress at the bottom fibre $= \frac{P}{A} - \frac{M}{I}y$.

By keeping $\frac{P}{A}$ more than $\frac{M}{Z}$, there will be no net tensile stresses in the section and the cracking is minimized.

Materials used in pre-stressed concrete.

1. Steel

- Ordinary mild steel and deformed bars used in RCC are not used in PSC (Pre-stressed concrete).
- Because their yield strength is not very high. In the PSC loss of pre-stress (about 20%) occurs due to many factors.
- If mild steel or HYSD bars are used, then very little pre-stress will be left after the losses and will be of no use for us.
- Therefore, high tensile strength steel is used for pre-stressing.
- In addition to the high strength, the steel used in pre-stressing must have a higher ultimate elongation.
- Various forms of steel used for pre-stressing are as follows.

a) Tendons.

- Tendons are high strength tensile wires available in various diameter from 1.5mm to 8mm.

Diameter of wire (mm)	Ultimate tensile strength (N/mm²)
1.5	2350
2.0	2200
3.0	1900
4.0	1750
5.0	1600
7.0	1500
8	1500

b) Wire strands or cables.

- A strand or cable is made of a bundle of wires spun together.
- The overall diameter of a cable is from 7 to 17 mm.

They are used for post-tensioning system.

c) Bars.

- High tensile steel bars of diameter 10mm or more are also used in pre-stressed concrete.

Concrete.

- Since high tensile steel is used in PSC, the concrete used should also be of good quality and high strength
- Therefore IS code recommends a minimum mix of M40 for pre-tensioned system and M30 for post-tensioned system.
- These mixes have high strength and high value of modulus of elasticity of concrete which result in less deflection :
- The concrete used in PSC should be well compacted
- High strength concrete is used in PSC for following reasons.

i) Use of high strength concrete results in smaller section.

ii) High strength concrete offers high resistance in tension, shear, bond and bearing.

iii) Less loss of pre-stress occurs with high strength concrete.

Advantages and disadvantages of pre-stress concrete.

Advantages of pre-stress concrete.

- Pre-stress concrete sections are thinner and lighter than RCC section since high strength concrete and steel are used in PSC.
- In PSC, the whole concrete area is effective in resisting loads unlike RCC. where concrete below the neutral axis is neglected
- Thinner sections in PSC results in less selfweight and hence overall economy.
- Long span bridges and flyovers are made of PSC.

because of lesser self weight and thinner section. So PSC is used for heavily loaded structures.

- Pre-stressed concrete members show less deflection
- Since the concrete doesnot crack in PSC, ~~resting~~ rusting of steel is minimized
- Pre-stressed concrete is used in the structure where tension develop or the structure is subjected to vibration, impact and ~~so~~ shocks like girders, bridges, railway sleepers, electric poles, gravity dam etc.
- Pre-cast members like electric poles and railway sleepers are produced easily in factories using ~~simple~~ single prestressing method.

Disadvantages.

i) Pre-stressed concrete construction require very good quality control and super-vision

ii) Cost of material used in PSC is very high (high tensile steel is about 3 times costlier than mild steel).

iii) PSC requires specialized tensioning equipments and devices which are very costly.

iv) Pre-stressed sections are more brittle because of use of high-tension steel

Pre-stressing methods.

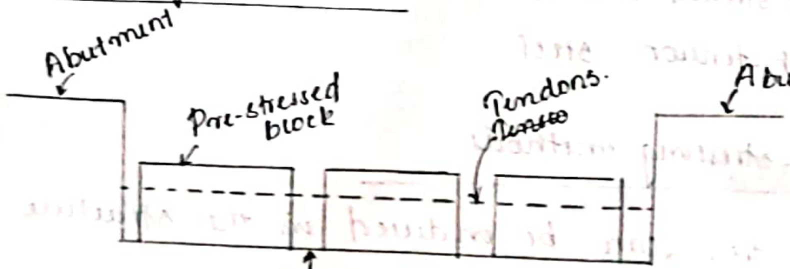
- Pre-stress can be induced in the structure by various pre-stressing methods which are:

1. Internal pre-stressing

- In this system, a pre-stressing force is applied to the high tensile steel i.e, the steel reinforcement. It induces internal compressive stresses in concrete.
- It is the most commonly used method because of easy and accurate application.

- Internal pre-stressing can be done by two methods
a) Pre-tensioning
b) Post-tensioning

a) Pre-tensioning method.



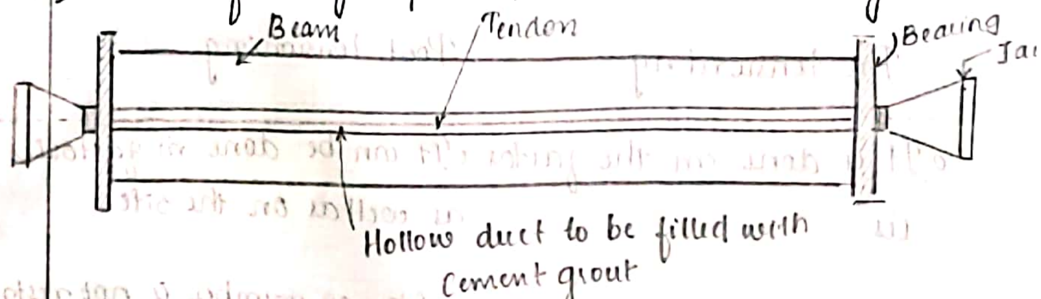
- In this method, pre-stress is induced (tendons are tensioned) before the concrete is placed.
- It is done in factories.
- In this method, the tendons are enclosed temporarily against some abutments and then they are pulled by using jack type devices.
- Concrete is placed while maintaining the tension.
- When concrete is hardened sufficiently, the tendons are released slowly or cut.
- This will transfer pre-stress from steel to concrete through bond.
- This method is commonly used for small sized members like beams, slabs, piles, sleepers and electric poles etc. which can be cast easily in factories.

Advantages and disadvantages of pre-tensioning.

- Pre-tensioning is done in the factories so it is more reliable and durable technique.
- But it is used for smaller sections, so heavier and longer sections cannot be pre-stressed.
- When cable is released or cut, after pre-tensioning then it leads to more losses due to shortening.
- The shrinkage and creep losses are also more in pre-tensioning system.

b) **Post tensioning method.**

- In this method, the pre-stress is induced or tendons are tensioned only after the concrete has hardened.
- In this system, the concreting is done first and the duct is formed in the member with tube or with a ~~material~~ metal sheathes.
- When concrete has sufficiently hardened, then tendon or cable is passed through the duct.
- It is tensioned and anchored at its ends.
- Pre-stressing force is transferred from the tendon to the member through anchorage wedges.
- The space b/w the tendon and the duct is filled with cement grout.
- Post tensioning method of prestressing is used for both pre-cast and cast-in situ construction.
- It is used for large spanned structures like bridges.



[Post tensioning method]

Advantages and disadvantages of post-tensioning method

i) Post tensioning can be done in factories and at the site also.

ii) The loss of pre-stress is less as compared to pre-tensioning system.

iii) This method is used for large spans and heavily loaded structures.

- The disadvantages of post-tensioning method is that it is costly as compared to pre-tensioning method because of use of sheathing.

Difference between pre-tensioning and post tensioning system.

Pre-tensioning	Post-tensioning
It is done in the factories	It can be done in factories as well as on the site.
Small sections are to be constructed	Size of member is not restricted so long span bridges are constructed by post tensioning.
Loss of pre-stress is more (About 18%)	Loss of pre-stress is less (About 15%)
It is cheaper because the cost of sheathing is not involved.	It is costlier because of use of sheathing
It is more reliable and durable.	The durability depends upon the two anchorage mechanism.

Pre-stressing systems.

- A pre-stressing system consist of stressing steel and a method of anchoring to the concrete.

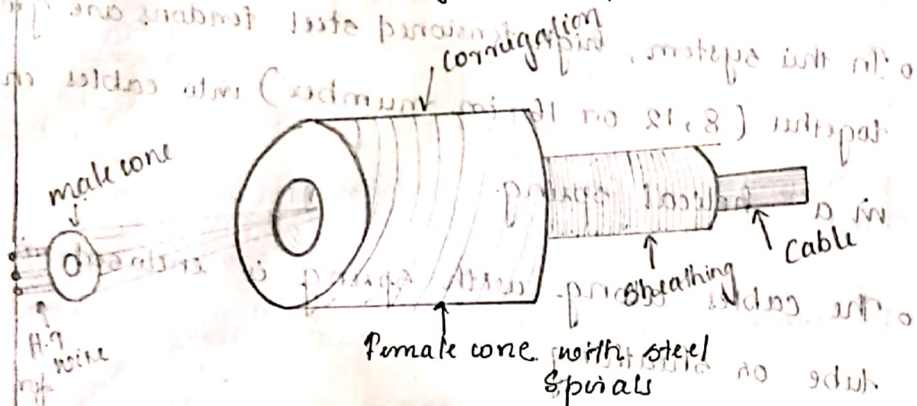
Some of the commonly used pre-stressing systems are as follows:

1. Freyssinet system.

- This system was developed by French Engineer Freyssinet and is most commonly used for post-tensioning.
- In this system, high tensioned steel tendons are grouped together (8, 12 or 16 in number) into cables enclosed in a helical spring.
- The cables along with spring is enclosed in a tube or sheathing.
- The cables projects out of to at the ends for of about 80mm for providing grip for tensioning
- The anchorage consist of a concrete cylinder with corrugations and having a conical hole.
- This cylinder is called as female cone.
- The conical plugs, called as male cones are pushed into the cylinder and cement is pumped through the hole to grout the space.

It prevents the wires from slipping.

- This method is not very costly as the wires can be secured easily and plugs can be left in the concrete as they donot project outside.

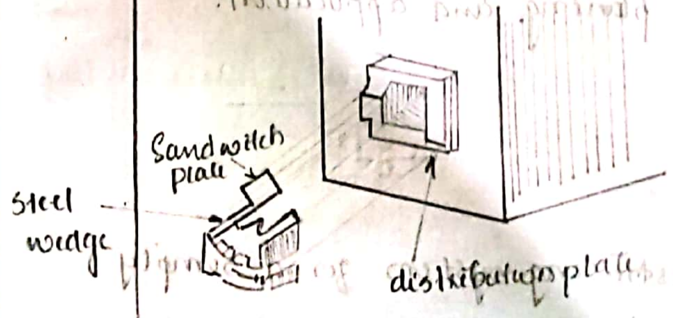


Magnel Blaton system.

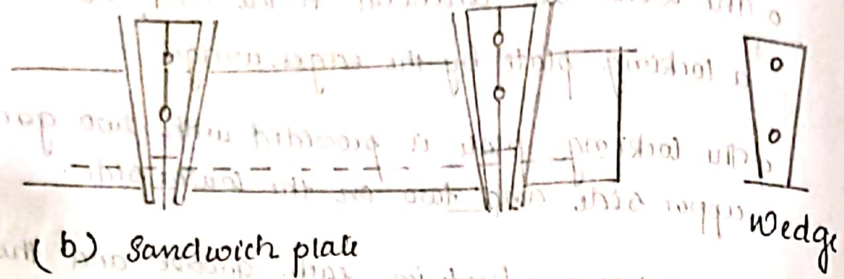
- In this system, an even no. of wires upto 64 of high tensile steel (5–7 mm diameter) are arranged in a group of four with the help of vertical and horizontal spacers.
- These spacers help the wires to remain in position.

- The wires are anchored at the end, two at a time into a locking plate by the edges. wedges.
- The locking plate is provided with two grooves on the upper side and two on the lower side.
- Two wires are kept in each groove and the steel wedge is inserted into the groove, thus tightening the wire.
- 8 wires are can be anchored on one locking plate and the locking plates are arranged one over the other.
- For prestressing wires, ducts are cast into the member by the moulds having rubber corks cores.
- The rubber cores are taken out after 8 – 10 hrs hours of concreting and duct are formed.

Magnel Blaton system.



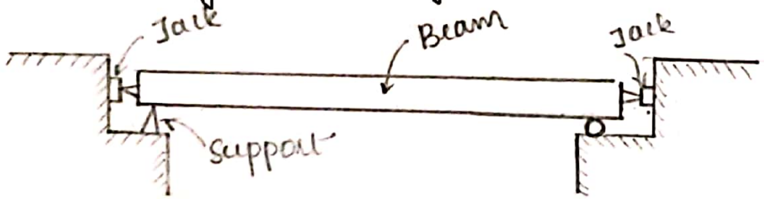
(a)



(b) Sandwich plate

External prestressing.

- This method is not commonly use.
- In this method, prestressing is done by adjusting the external reaction (by introducing different support condition)
- A Simply supported beam is prestressed externally by jacks that shown in figure.
- The externally prestressing system requires very much accuracy in planing and application.



Externally prestressing system for a simply supported beam

Losses of prestress

- The applied prestressing force doesnot remainconstant but decreases with time.
- The amount of force which gets reduced or lost is known as loss of prestress
- It may vary from 15% to 20%.
- This loss of prestress is due to the following reasons:

i) Loss due to elastic shortening :

- In pretensioned beams, when the pre-stress is transferred from steel to concrete, the concrete member gets shortened along with steel.
- This results in loss of prestress which may range from 3-6%

[Page no. 32 , IS-1343-1980, cl-18.5.2.]

$$\varepsilon_c = \frac{f_c}{E_c}$$

$$\varepsilon_s = \frac{f_s}{E_s}$$

ε_c = Strain in concrete
ε_s = " " steel.
E_c = Young's modulus of con
E_s = " " steel
f_c = stress in concrete
f_s = " " steel.

$$\frac{f_c}{E_c} = \frac{f_s}{E_s}$$

$$f_s = \frac{E_s}{E_c} f_c$$

$$f_s = m \times f_c$$

where,

ε_c = strain in concrete at the center of cable

ε_s = strain in steel at the center of cable

f_s and f_c = loss of stress at the level of center of cables in steel and concrete respectively

m = modular ratio $\left(m = \frac{E_s}{E_c}\right)$

ii) Loss due to creep of concrete.

- Creep is the strain caused in concrete due to sustained (constant) stress over a period of long time, in PSC, the sustained stress is the pre-stress which causes creep shortening.
- Pre stressed members have more creep loss than post tensioned because they are pre-stressed at earlier (before concreting)
- This loss of pre stress may range from 5-10%.
- Loss of pre stress due to creep of concrete = ~~$E_s \times \varepsilon_{ul}$~~

$= E_s \times$ ultimate creep strain in concrete at the centroid of section.

$= E_s \times \varepsilon_{uc}$

where,

ε_{uc} = creep coefficient (ϕ) x initial elastic strain

- In the absence of actual data creep coefficients can be taken from IS 456-2000 Cl : 6.2.5.1
- $\therefore \varepsilon_{uc} = \phi \times \frac{f_c}{E_c}$ → Strain

where, f_c = sustained ~~strain~~ stress

E_c = Young's modulus of concrete.

$\therefore$ Loss of pre-stress due to creep $= E_s \times \phi \times \frac{f_c}{E_c}$.

$= m \times \phi \times f_c$.

$$m = \frac{E_s}{E_c}$$

iii) Loss due to shrinkage of concrete.

- Concrete shrinks because of drying and chemical changes.

- The shrinkage in concrete depends upon the quantity of water, aggregate, atmospheric condition and time.
- The loss of pre-stress due to shrinkage is calculated as follows:

- Loss of pre stress $= E_s \times \varepsilon_{sh}$

  where
  E_s = modulus of elasticity of steel
  ε_{sh} = shrinkage strain

- For pre tensioning, $\varepsilon_{sh} = 0.0003$

  For post tensioning $\varepsilon_{sh} = \frac{0.0002}{\log_{10}(t+2)}$

  where,
  t = age of concrete in days.

- For pre tensioned members, this loss is about 5% and for post tensioned it is about 3-4%.

v) Loss due to relaxation of stress

- Loss of pre stress occurs due to relaxation, under a constant strain.
- It is time dependent and also depends upon the amount of stress.
- This loss is about 2-8% of pre-stress.

v) Loss due to friction.

- Friction loss occurs in post-tensioned members only, due to friction in jacks between the tendons and ducts or spaces etc.
- This loss can be reduced by lubricating the cables, applying pre stress from both ends and avoiding large curvatures etc.
- Due to curvature effect $P_x = P_0 e^{-\mu\alpha}$
- Similarly due to wave effect $P_x = P_0 e^{-kx}$
- Combining the effect of curvature and wave on the pre stress $P_x = P_0 (e^{-(\mu\alpha + kx)})$

  where, P_x = Pre stressing force at a distance x from the jack end in the direction of tangent to the curve

P_0 = Prestressing force at the jack end in the direction of tangent to the curve.

μ = Coefficient of friction (given in code IS 1343 - 1980)

α = Angle of curvature in radiants of the cable (given in code IS 1343 - 1980)

k = Coefficient for wave effect as given in code IS 1343 - 1980)

vi) **Loss due to slip.**

- When the pre stress is transferred from steel tendons to concrete, the tendons may slip causing loss of pre stress.
- This slip may vary from 2mm - 5mm
- The slip can be calculated as follows

$$\delta_s = \frac{P_1 L}{A E_s}$$

where δ_s = slip

P_1 = ~~loss prestress~~ losed prestressing force

L = Length of cable.

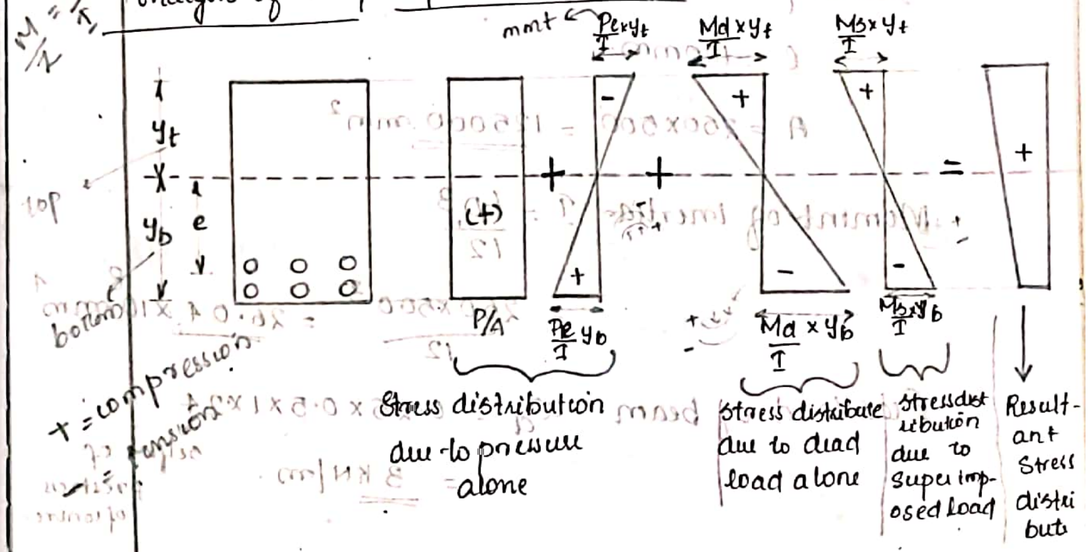
A = cross sectional area of cables.

E_s = Young's modulus of steel &

- Knowing the value of slip (δ_s), the pre stress lost (P_1) due to slip can be calculated.

$\frac{M}{I} = \frac{My}{I}$

Analysis of simple prestressed ~~recta~~ rectangular section.



1 Determine the various stresses setup at mid span in a pretensioned beam 250mm x 500mm, subjected to an initial pre stress 1500kN and a uniformely distributed super imposed load of 5kN/m over

P_0 = Prestressing force at the jack end in the direction of tangent to the curve.

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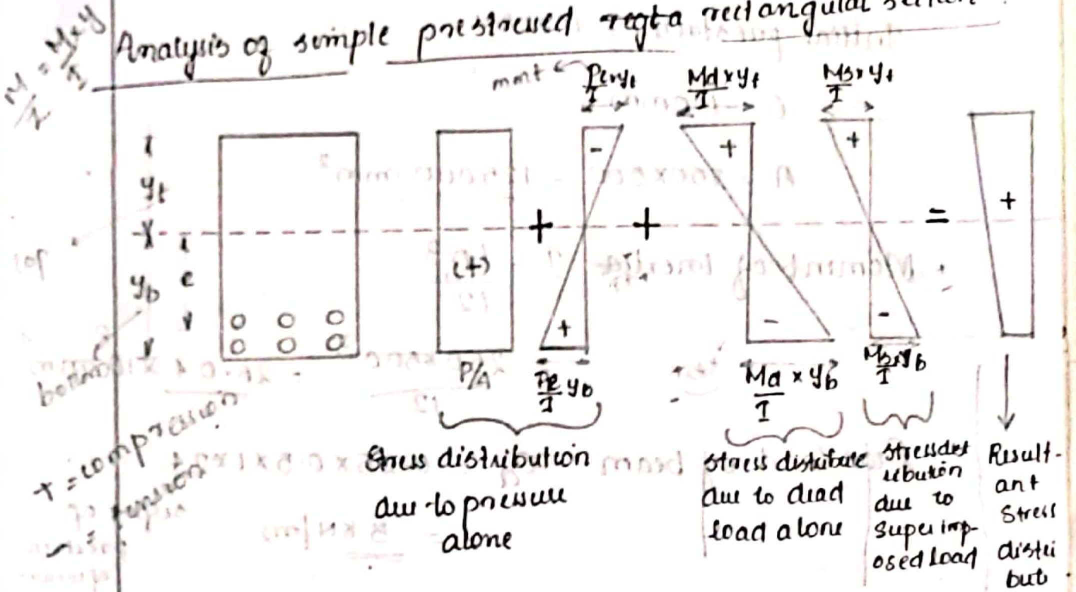
- Knowing the value of slip (δ_s), the pre stress lost (P_1) due to slip can be calculated.

Analysis of simple prestressed ~~recta~~ rectangular section



1 Determine the various stresses set up at mid span in a pretensioned beam 250mm x 500mm, subjected to an initial pre stress 1500kN and a uniformly distributed super imposed load of 5kN/m over

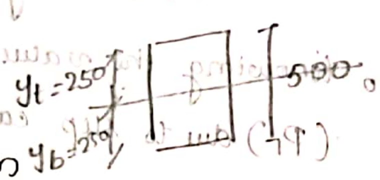
a span of 15m. Assume total loss of pre stress as 12% and eccentricity of pre stress at mid span is 100mm.

Given data,

$b = 250mm$

$D = 500mm$

$L = 15m$.



Super imposed load = 5 kN/m

Initial pre stress = 1500kN

$e = 100mm$.

$A = 250 \times 500 = \underline{125000 \; mm^2}$

Moment of Inertia $I = \frac{bD^3}{12}$

$= \frac{250 \times 500^3}{12} = \underline{26.04 \times 10^8 \; mm^4}$

Dead load of beam $w_d = 0.25 \times 0.5 \times 1 \times 24$ (self wt of prestress of concrete)

$= \underline{3 \; kN/m}$.

Moment due to dead load $M_d = \frac{w_d l^2}{8}$

$= \frac{3 \times 15^2}{8}$

$= \underline{84.375 \; kNm}$

Moment due to super imposed load $= M_s = \frac{w_s l^2}{8}$

$= \frac{~~1500~~ \times 15^2 \times 5}{8}$

$= \underline{140.625 \; kNm}$

1. Calculation of stresses at initial stage.

- Stress at the top most concrete fibre $= \frac{P_i}{A} - \frac{P_i \times e}{I} \times y_t + \frac{M_d}{I} \times y_t$.

$= \frac{1500 \times 10^3}{125000} - \frac{1500 \times 10^3 \times 100}{26.04 \times 10^8} \times 250 + \frac{84.375 \times 10^6 \times 250}{26.04 \times 10^8}$

$= 12 - 14.4 + 8.1 = \underline{5.7 \; N/mm^2}$ (compressive)

- Stress at bottom most fibre $= \frac{P_i}{A} + \frac{P_i e}{I} \times y_b - \frac{M_d}{I} y_b$

$= \frac{1500 \times 10^3}{125000} + \frac{1500 \times 10^3 \times 100 \times 250}{26.04 \times 10^8} - \frac{84.375 \times 10^6 \times 250}{26.04 \times 10^8}$

$= 12 + 14.4 - 8.1 = \underline{18.3 \; N/mm^2}$ (compressive)

2. At final stage i.e, after transfer of pre stress and under the action of super imposed loads.

i) Stress at top fibre in concrete $= \frac{P_e}{A} - \frac{P_e}{I} y_t + \frac{M_d}{I} y_t + \frac{M_s}{I} y_t$

[P_e = effective prestress]

Loss →

Considering 12% loss of pre-stress

Effective prestress $P_e = \frac{100-12}{100} \times 1000$

$= 1320 \text{ kN}$.

Stress at top most fibre $= \frac{P_e}{A} - \frac{P_e e}{I} y_t + \frac{M_a y_t}{I} + \frac{M_b y_t}{I}$

$= \frac{1320\times10^3}{125000} - \frac{1320\times10^3 \times 100\times250}{26.04\times10^8} + \frac{84.375\times10^6\times250}{26.04\times10^8}$

$+ \frac{140.625\times10^6 \times 250}{26.04\times10^8}$

$= 19.48 \text{ N/mm}^2$ (compression)

Stress at bottom most fibre $= \frac{P_e}{A} + \frac{P_e}{I} y_b - \frac{M_a}{I} y_b$

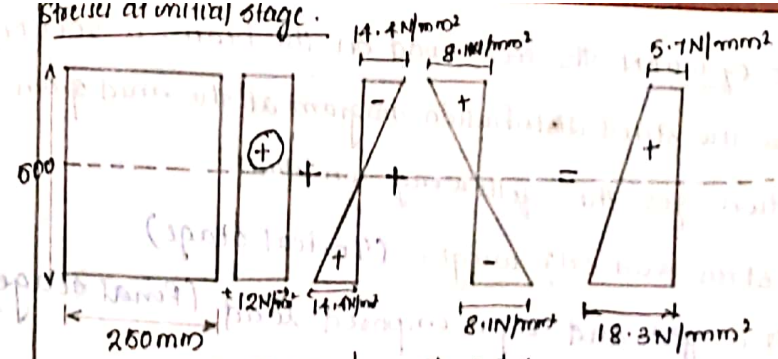
$- \frac{M_b}{I} y_b$.

$= \frac{1320\times10^3}{125000} + \frac{1320\times10^3\times100\times250}{26.04\times10^8} - \frac{84.375\times10^6\times250}{26.04\times10^8}$

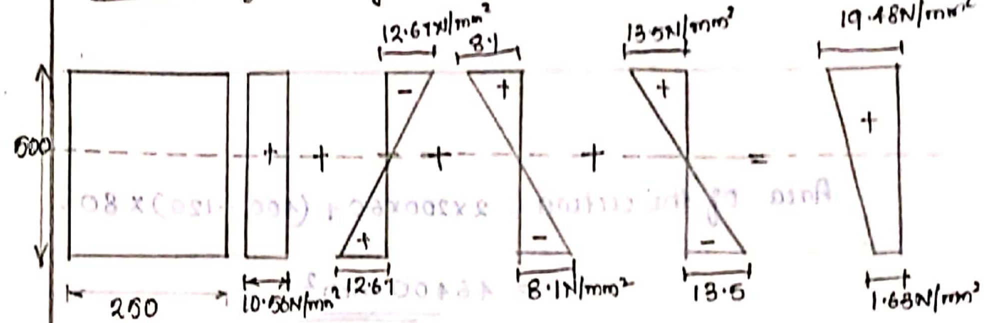
$- \frac{140.625\times10^6\times250}{26.04\times10^8}$

$= 1.63 \text{ N/mm}^2$ (compressive)

Stresses at initial stage.



Stresses at final stage.



2 A beam of symmetrical I section spanning 8m has a flange width of 200mm and a flange thickness of 60mm respectively. The overall depth of the beam is 400mm. Thickness of web is 80mm. The beam is prestressed by a parabolic cable with an eccentricity of 150mm at the centre and 0 at supports with an effective prestressing

$P_{loss} \rightarrow$

Considering 12% loss of pre-stress

Effective prestress $P_e = \frac{100-12}{100} \times 1000$

$= \underline{\underline{1320 \text{ kN}}}$.

Stress at top most fibre $= \frac{P_e}{A} - \frac{P_e e}{I} y_t + \frac{M_d y_t}{I} + \frac{M_s y_t}{I}$.

$= \frac{1320 \times 10^3}{125000} - \frac{1320 \times 10^3 \times 100 \times 250}{26.04 \times 10^8} + \frac{84.375 \times 10^6 \times 250}{26.04 \times 10^8}$

$+ \frac{140.625 \times 10^6 \times 250}{26.04 \times 10^8}$

$= \underline{\underline{19.48 \text{ N/mm}^2}}$ (compression)

Stress at bottom most fibre $= \frac{P_e}{A} + \frac{P_e e}{I} y_b - \frac{M_d}{I} y_b$

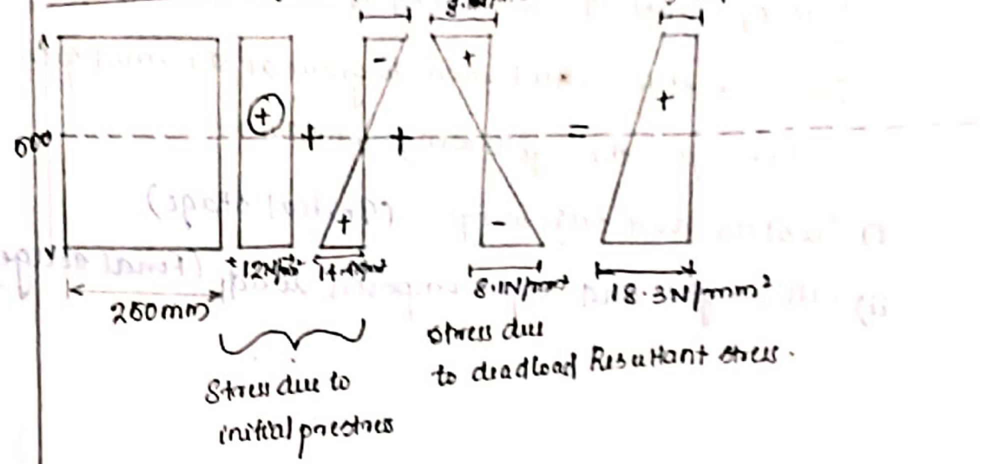
$- \frac{M_s}{I} y_b$.

$= \frac{1320 \times 10^3}{125000} + \frac{1320 \times 10^3 \times 100 \times 250}{26.04 \times 10^8} - \frac{84.375 \times 10^6 \times 250}{26.04 \times 10^8}$

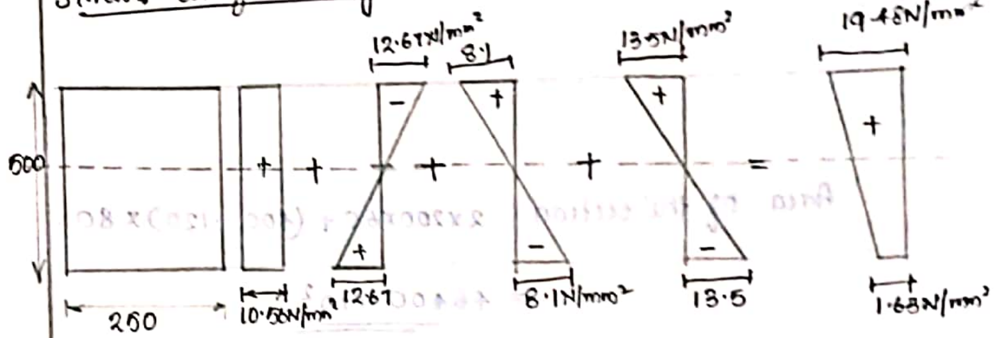
$- \frac{140.625 \times 10^6 \times 250}{26.04 \times 10^8}$.

$= \underline{\underline{1.68 \text{ N/mm}^2}}$ (compressive)

Stresses at initial stage.



Stresses at final stage.

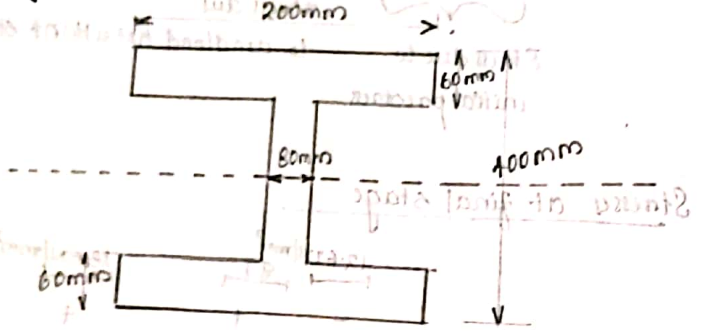


2 A beam of symmetrical I section spanning 8m has a flange width of 200mm and a flange thickness of 60mm respectively. The overall depth of the beam is 400mm. Thickness of web is 80mm. The beam is prestressed by a parabolic cable with an eccentricity of 150mm at the centre and 0 at supports with an effective prestressing

force of 100 kN. The live load on the beam is 2000 N/m. Draw the stress distribution diagram at the mid span section for the following condition

i) Prestress and self weight (Initial stage)

ii) Self weight and super imposed load (Final stage)



Area of the section $= 2\times200\times60 + (400-120)\times 80$

$= 46400\ mm^2$

Moment of inertia of the section $= \frac{200\times400^3}{12} - 2\left[\frac{60\times280^3}{12}\right]$

$= 8.47\times10^8\ mm^4$

Prestress and self weight (Initial stage)

Self weight of the beam $= 24\times46400\times10^{-6}\times1$

$= 1.1136\ kN/m$

Moment due to selfweight at mid span $= \frac{W_d\times l^2}{8}$

$= \frac{1.1136\times8^2}{8}$

$= 8.9088\ kNm$

Effective prestress $= 100\ kN$.

Assuming 15% loss

Stresses at top fibre $= \frac{P_i}{A} - \frac{P_i e}{I}y_t + \frac{M_d}{I}y_t$

$= \frac{100\times10^3}{46400} - \frac{100\times10^3\times150\times200}{8.47\times10^8} + \frac{8.908\times10^6\times200}{8.47\times10^8}$

Initial prestress $= \frac{100\times100}{100-15}$

$= 117.647\ kN$.

$\therefore$ Stresses at top fibre $= \frac{117.647\times10^3}{46400} - \frac{117.647\times10^3\times150\times200}{8.47\times10^8} + \frac{8.908\times10^6\times200}{8.47\times10^8}$

$= 0.47196\ N/mm^2$.

Stresses at bottom fibre $= \frac{P_i}{A} + \frac{P_i e}{I}y_b - \frac{M_d}{I}y_b$

$= \frac{117.647\times10^3}{46400} + \frac{117.647\times10^3\times150\times200}{8.47\times10^8} - \frac{8.908\times10^6\times200}{8.47\times10^8}$

$= 4.599\ N/mm^2$

Prestress + self weight + live load.

Super imposed load $w = 2000 N/m$

$= 2 kN/m$

Moment due to live load $= \frac{wl^2}{8}$

M_s

$= \frac{2 \times 8^2}{8} = 16 kNm$

~~Stresses~~ Effective pre stress $= 100 kN$.

Stresses at top $= \frac{P_e}{A} - \frac{P_e \times e \times y_t}{I} + \frac{M_d}{I} y_t + \frac{M_s}{I} y_t$.

$= \frac{100 \times 10^3}{46400} - \frac{100 \times 10^3 \times 150 \times 200}{8.47 \times 10^8} + \frac{8.9088 \times 10^6 \times 200}{8.47 \times 10^8} + \frac{16 \times 10^6 \times 200}{8.47 \times 10^8}$

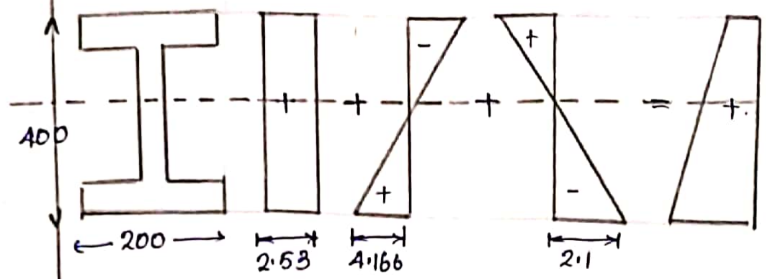
$= 4.4949 N/mm^2$

Stresses at bottom $= \frac{P_e}{A} + \frac{P_e \times e \times y_b}{I} - \frac{M_d}{I} y_b - \frac{M_s}{I} y_b$

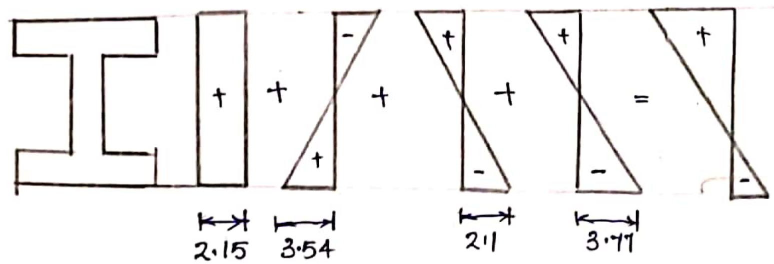
$= \frac{100 \times 10^3}{46400} + \frac{100 \times 10^3 \times 150 \times 200}{8.47 \times 10^8} - \frac{8.9088 \times 10^6 \times 200}{8.47 \times 10^8} + \frac{16 \times 10^6 \times 200}{8.47 \times 10^8}$

$= -0.1845 N/mm^2$

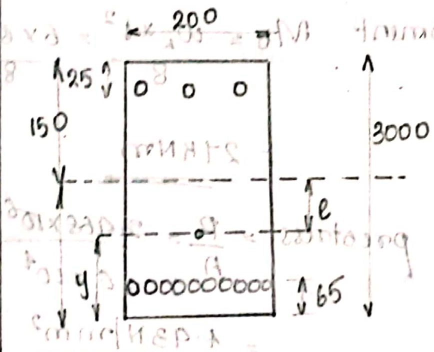
Stresses at initial stage.

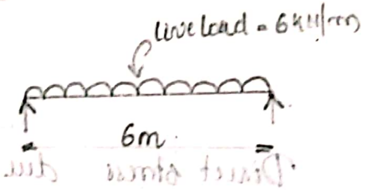


Stresses at final stage.



3 A rectangular beam of cross section 30cm deep and 20cm wide is prestressed by means of 15 wires of 5mm diameter located 6.5cm from the bottom of the beam and the 3 prewires of diameter of 5mm, 2.5cm from the top. Assuming the prestress in the steel as 840N/mm², calculate the stresses at the extreme fibres of the mid span section when the beam is supporting its own weight over a span of 6m. If a uniformly distributed live load of 6kN/m is imposed, evaluate the working stress in concrete. The density of concrete is 24kN/m².





Distance of the centroid of the prestressing force from the base, $(y)_{A_1} = \frac{15 \times 65 + 3 \times 275}{18} = \underline{\underline{100\,mm}}$

Eccentricity $e = 1500 - 100 = 60mm$

Prestressing force , $P = 840 \times 18 \times \frac{\pi}{4} \times 5^2 = 296880.60$

$\simeq 2.9 \times 10^5 N$

Area of c/s , $A = 300 \times 200 = 6 \times 10^4 mm^2$

Moment of inertia , $I = \frac{bd^3}{12} = \frac{200 \times 300^3}{12} = 45 \times 10^7 mm^4$

Section modulus $Z = \frac{bd^2}{6} = \frac{200 \times 300^2}{6} = 3 \times 10^6 mm^3$

Self weight of the beam $= 0.3 \times 0.2 \times 24 = 1.44 kN/m$

Self weight moment , $M_d = \frac{wl^2}{8} = \frac{1.44 \times 6^2}{8} = 6.48 kNm$

Live load bending moment $M_s = \frac{w_L \times l^2}{8} = \frac{6 \times 6^2}{8}$

$= 27 kNm$

Direct stress due to prestress $= \frac{P}{A} = \frac{2.968 \times 10^5}{6 \times 10^4}$

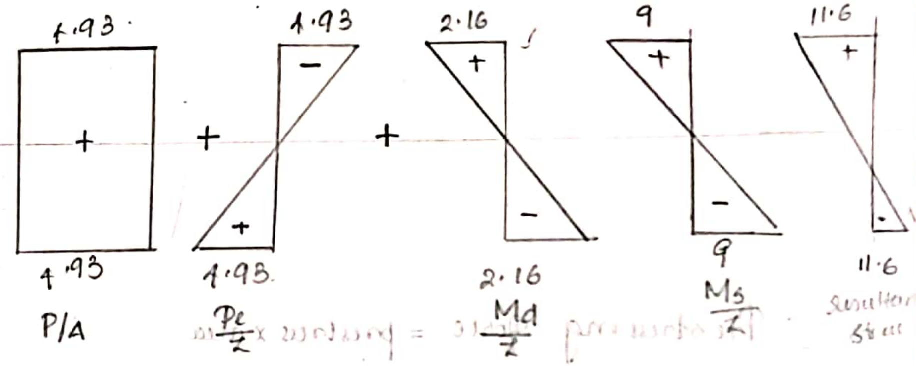
$= 4.93 N/mm^2$

Bending stress due to prestress $= \frac{Pe}{Z}$

$= \frac{2.96 \times 10^5 \times 60}{3 \times 10^6} = 4.93 N/mm^2$

Self weight stress $= \frac{M_d}{Z} = \frac{6.48 \times 10^6}{3 \times 10^6} = 2.16 N/mm^2$

The resultant stresses due to [self wt + prestress + live load] are shown in figure.



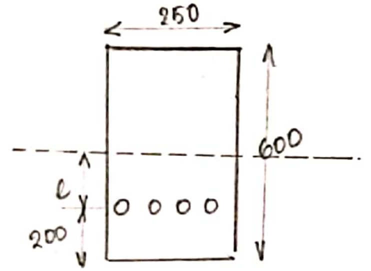
4 A rectangular concrete beam 250mm wide 600mm deep is prestressed by means of 4 nos 14mm diameter, high tensile bars located 200mm from the soffit of the beam. If the effective stress in the wire is 700 N/mm², which is the maximum BM that can be applied to the section without causing tension at soffit of the beam.

$A = 250 \times 600 = 15 \times 10^4 mm^2$

$z = \frac{bd^2}{6} = \frac{250 \times 600^2}{6} = 15 \times 10^6 \, mm^3$

Area of tendon $= 4 \times \frac{\pi}{4} \times (14)^2 = 615.75 \, mm^2$

Eccentricity $e = 300 - 200 = 100 mm$.



Prestressing force = prestress x area.

$= 700 \times 615.75 = 431025 N$.

Direct stress due to pressure $= \frac{P}{A} = \frac{431025}{15 \times 10^4} = 2.87 N/mm^2$

Bending stress due to pressure $= \frac{Pe}{z}$

Prestress at the soffit of the beam $= 2.87 + 2.87$

$= 5.74 N/mm^2$

If M = Maximum moment of the section of zero tension at the bottom face

$\frac{M}{z} = 5.73$

$M = 5.73 \times 15 \times 10^6 = 85.95 kNm$

5 A prestressed beam of section 200mm wide and 300mm deep is used over an effective span of 6m to support an imposed load of 4 kN. The density of concrete is 24 kN/m² at the centre of span section of the beam find the magnitude of :

a) The concentric prestressed force necessary for zero fibre stress at the soffit when the beam is fully loaded.

b) The eccentric prestressing force located 100mm from the bottom of the beam which would nullify the bottom fibre stress due to loading.

$A = 200 \times 300 = 6 \times 10^4 \, mm^2$

$z_b = z_t = \frac{bd^2}{6} = \frac{200 \times 300^2}{6} = 3 \times 10^6 \, mm^3$

Dead load $= 0.2 \times 0.3 \times 24 = 1.44 \, kN/m$

Moment due to dead load, $M_d = \frac{wl^2}{8} = \frac{1.44 \times 6^2}{8}$

$= 6.48\ kNm$

Moment due to super imposed load $M_s = \frac{wl^2}{8}$

$= \frac{4 \times 6^2}{8} = 18\ kNm$

○ Tensile stress at the bottom fibre due to dead and live load $= \frac{M_d}{Z} + \frac{M_s}{Z}$

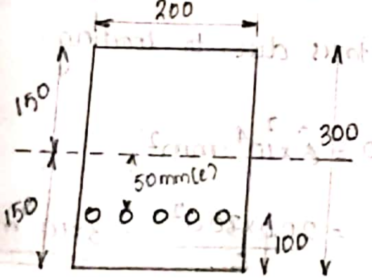
$= \frac{18 + 6.48}{3 \times 10^6} = 8.16\ N/mm^2$

a) If P = concentric prestressing force, for zero stress at the soffit of the beam under loads

$P/A = 8.16$

$P = 8.16 \times 6 \times 10^4 = 489.6\ kN$

b) If P = eccentric prestressing force $(e = 50m)$



For zero stress at the soffit of the beam under the loads

$$\frac{P}{A} + \frac{Pe}{Z_b} = \frac{M}{Z}.$$

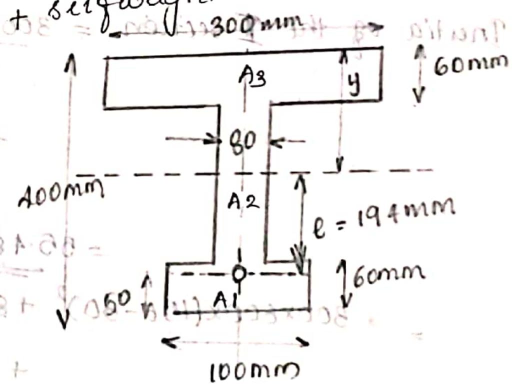
$$P \left[\frac{1}{6 \times 10^4} + \frac{50}{3 \times 10^6} \right] = 8.16$$

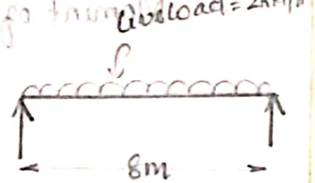
$P = 244.8\ kN$

6 An unsymmetrical I section beam is used to support an imposed load of 2kN/m over a span of 8m. The section details are top flange, 300mm wide and 60mm thick, bottom flange 100mm wide and 60mm thick. Thickness of the web = 80mm, Overall depth of the beam = 400mm. At the centre of the span, the effective prestressing force of 100kN is located at 50mm from the sockpit of the beam. Estimate the stresses at the centre of span section of the beam for following load condition.

i) Prestress + ~~live~~ self weight.

ii) Prestress + self weight + live load.





Prestressing force $P = 100\,kN$.

Area of concrete $A = A_1 + A_2 + A_3$

$= 300 \times 60 + 60 \times 100 + 80 \times 280$

$= \underline{46400\ mm^2}$

Distance of centroid from top $y = \dfrac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3}$

$= \dfrac{300 \times 60 \times 30 + 60 \times 100 \times 370 + 80 \times 280 \times 200}{46400}$

$= \underline{156.034\ mm}$.

$\therefore\ e = 400 - 156.034 - 50$

$= \underline{194\ mm}$.

Moment of Inertia of the I section $= \dfrac{300 \times 60^3}{12} + \dfrac{80 \times 280^3}{12} + \dfrac{100 \times 60^3}{12}$

$= 55.48 \times 10^8\ mm^4$.

$= 300 \times 600 \times (156-30)^2 + 80 \times 280 \times (200-156)^2 + 60 \times 100 \times (370-156)^2$

$= \underline{75.74 \times 10^7\ mm^4}$

Section modulus at top $Z_t = \dfrac{I}{y_t} = \dfrac{75.74 \times 10^7}{156}$

$= \underline{485 \times 10^4\ mm^3}$.

Section modulus at bottom

$Z_b = \dfrac{I}{y_b} = \dfrac{75.74 \times 10^7}{244} = \underline{310.41 \times 10^4\ mm^3}$

Dead load of the I section = Area x self weight

$= 46400 \times 24\,kN/m^3 \times 10^{-6}$

$= 1113600\ kN \times 10^{-6}$

$= \underline{1.11\ kN/m}$.

Moment due to dead load $M_d = \dfrac{wl^2}{8}$

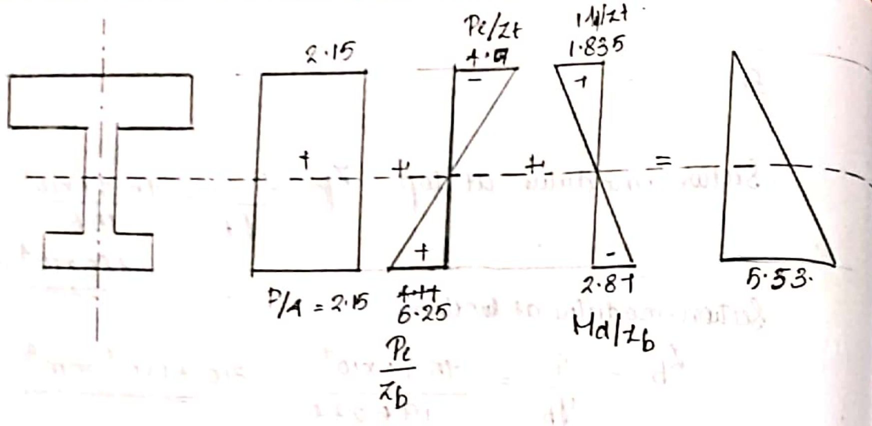
$= \dfrac{1.11 \times 8^2}{8} = \underline{8.9088\ kNm}$.

Moment due to imposed load $M_s = \dfrac{wl^2}{8}$

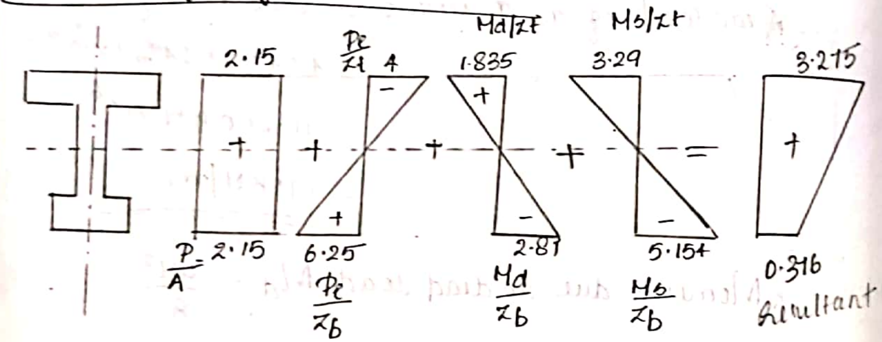
$= \dfrac{2 \times 8^2}{8} = \underline{16\ kNm}$

Stresses at the centre of span.

a) Prestress + Self weight



b) Prestress + self weight + live load.



Losses in prestress.

7 Determine the % of total loss of prestress in a simply supported pretensioned beam of size 150x300mm. having 8 wires of 6mm diameter subjected to an ~~center~~ initial prestress of $1000 N/mm^2$ at an eccentricity of 50mm. Take the following data for calculation of losses.

Creep coefficient = 1.6

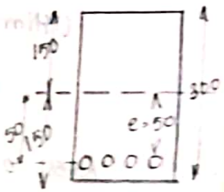
Loss due to relaxation = 5%

Use M40 concrete, $E_s = 2 \times 10^5 N/mm^2$

Solution

Cross sectional area of beam $A = 150 \times 300 = \underline{45000 mm^2}$

Moment of inertia $= \frac{bd^3}{12} = \frac{150 \times 300^3}{12} = \underline{3.375 \times 10^8 mm^2}$



$E_c = 5000\sqrt{f_{ck}}$

$= 5000\sqrt{40} = 31622.7766 N/mm^2$

$m = \frac{E_s}{E_c} = \frac{2 \times 10^5}{31622.7766} = \underline{6.3245}$

Area of steel wire $= A_s = 8 \times \frac{\pi}{4} \times (6)^2 = \underline{226.19 mm^2}$

Initial prestressing force P_I = stress x area

$= 1000 \times 226.19$

$= \underline{226194.67 N}$

$= \underline{226.19 kN}$.

Stress in concrete at the level of steel $= \frac{P_I}{A} + \frac{P_I \times e \times y}{I}$

$= \frac{226.19 \times 10^3}{45000} + \frac{226.19 \times 10^3 \times 50 \times 50}{3.375 \times 10^8}$ (y = e at the level of steel)

$= \underline{6.7 N/mm^2}$

i) Loss of prestress due to elastic shortening of concrete

$= m \times f_c$

$= 6.3245 \times 6.7 = \underline{\underline{42.387\ N/mm^2}}$

ii) Loss due to creep of concrete $= m \times \phi \times f_c$.

$= 6.3245 \times 1.6 \times 6.7 = \underline{\underline{67.79\ N/mm^2}}$

iii) Loss of prestress due to shrinkage of concrete

Ultimate shrinkage strain in concrete $(\varepsilon_{sh}) = 0.0003$ for pre-tensioned beams

Loss of prestress due to shrinkage of concrete

$= \varepsilon_{sh} \times E_s$.

$= 0.0003 \times 2 \times 10^5$

$= \underline{\underline{60\ N/mm^2}}$

iv) Loss due to relaxation of stress in steel = 5% of initial stress

$= \frac{5}{100} \times 1000 = \underline{\underline{50\ N/mm^2}}$

Total loss of prestress $= 42.387 + 67.79 + 60 + 50$

$= \underline{\underline{220.177\ N/mm^2}}$

% loss of prestress $= \frac{220.177}{1000} \times 100$

$= \underline{\underline{22.0177\ \%}}$.

8. A prestress concrete beam of rectangular cross section 120mm wide and 300mm deep is prestressed by 6 wires of 6mm diameter, provided at an eccentricity of 55mm. The initial prestress in the wire 1150 N/mm². Find the loss of stress due to creep of concrete. Take $E_s = 2 \times 10^5\ N/mm^2$, $E_c = 3 \times 10^4\ N/mm^2$ and creep coefficient of concrete as 1.5

$b = 120mm$

$D = 300mm$

$A_s = 6 \times \frac{\pi}{4} \times (6)^2 = \underline{169.64\ mm^2}$

Initial prestress $= \underline{\underline{1150\ N/mm^2}}$

$E_s = 2 \times 10^5\ N/mm^2$

$E_c = 3 \times 10^4\ N/mm^2$

Creep coefficient $\phi = 1.5$

Modular ratio $m = \frac{E_s}{E_c} = \frac{2 \times 10^5}{3 \times 10^4} = \underline{\underline{6.66}}$.

$e = 55\,mm$

$A = b \times D = 120 \times 300 = 36000\,mm^2$

Initial prestressing force P = stress $\times A_s$.

$= 1150 \times 169.64$

$= 195.086\,kN$.

Moment of inertia $I = \frac{bd^3}{12}$

$= \frac{120 \times 300^3}{12} = 2.7 \times 10^8\,mm^4$

Stress in concrete at the level of steel (f_c)

$= \frac{P_I}{A} + \frac{P_I \times e \times y}{A I}$ (Here $y = e$ at a level of steel)

$= \frac{195.086 \times 10^3}{36000} + \frac{195.086 \times 10^3 \times 55 \times 55}{2.7 \times 10^8}$

$= 7.6\,N/mm^2$

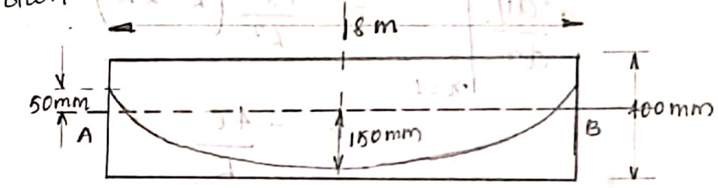
Loss due to creep of concrete $= \phi \times M \times m \times f_c$

$= 1.5 \times 6.66 \times 7.6$

$= 75.97\,N/mm^2$

% loss $= \frac{75.97}{1150} \times 100 = 6.6\,\%$

9 A post tensioned beam of size 200 x 400mm is prestressed by parabolic cables with 150mm eccentricity at centre of span and 50mm eccentricity at ends. The span of the beam is 18m. If the cables are having an area of 500mm² and subjected to an initial prestress of 1200N/mm². Determine the loss in prestress due to friction. Take $\mu = 0.25$ and $k = 15 \times 10^{-4}$ /m length.



$b = 200mm$.

$D = 400mm$

$A_s = 500mm^2$

Initial stress $= 1200\,N/mm^2$

$\mu = 0.25$

$k = 15 \times 10^{-4}$ /m length.

The equation of the cables parabolic profile with origin at a support

$y = \frac{4ex}{L^2}(L - x)$

$$\frac{dy}{dx} = \frac{4e}{L^2}\left[(L-x)+(-1)\times x\right]$$

$$= \frac{4e}{L^2}(L-2x).$$

Slope at $x = 0$,

$$\left.\frac{dy}{dx}\right|_{x=0} = \frac{4e}{L}.$$

Slope at $x = L$

$$\left.\frac{dy}{dx}\right|_{x=L} = \frac{4e}{L^2}(L-2L)$$

$$= -\frac{4e}{L}.$$

∴ Total change in slope from $x=0$ to $x=L$ [from A to B]

$$= \frac{4e}{L} - \left(-\frac{4e}{L}\right) = \frac{8e}{L}$$

Total eccentricity at support and at mid span (e)

$$= 50+150 = 200\,mm.$$

Total change in slope $(\alpha) = \frac{8e}{L}$

$$= \frac{8\times 200}{18\times 10^3} = 0.0888\,rad.$$

Initial prestress at the end A (P_A) $= 1200\times 500 = 600000\,N$

$= 600\,kN.$

Prestressing force at other end B (P_B) [Refer IS 1343-1980 page 33 Cl: 18.5.2.6]

$$P_B = P_A \times e^{-(\mu\alpha + kx)}$$

$$= 600 \times e^{-(0.25\times 0.088 + 15\times 10^{-4}\times 18)}$$

$$= 571.35\,kN.$$

Loss of prestress due to friction $= P_A - P_B$

$$= 600 - 571.35$$

$$= 28.65\,kN.$$

% Loss of prestress due to friction $= \frac{28.65}{600}\times 100$

$$= 4.775\,\%$$

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10 Calculate the % loss of prestress due to shrinkage of concrete prestressed by a cable carrying and initial prestress of 600N/mm². The prestress is transferred at the age of 7days. Assume the beam as

i) Pre-tensioned

ii) Post-tensioned.

Solution.

i) Pre-tensioned beam.

Shrinkage strain in concrete $\varepsilon_{sh} = 3\times10^{-4}$

∴ Loss of prestress $= \varepsilon_{sh}\times E_s$

$= 3\times10^{-4}\times2\times10^{5} = 60N/mm^2$.

$\%$ loss of prestress $= \frac{60}{600}\times100 = 10\%$

ii) Post-tensioned beam.

Shrinkage strain $\varepsilon_{sh} = \frac{2\times10^{-4}}{\log_{10}(t+2)}$

Here, t = 7days

$\therefore \varepsilon_{sh} = \frac{2\times10^{-4}}{\log_{10}(7+2)} = 2.0959\times10^{-4}$

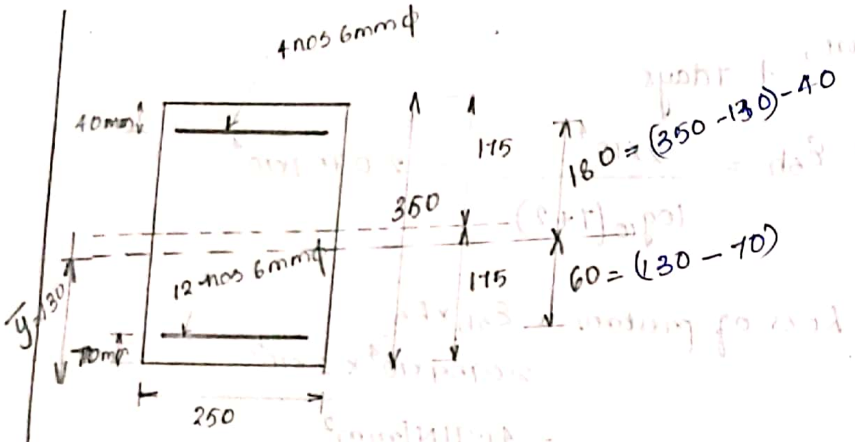
Loss of prestress $= \varepsilon_{sh}\times E_s$

$= 2.0959\times10^{-4}\times2\times10^{5}$

$= 41.91N/mm^2$

$\%$ loss of prestress $= \frac{41.91}{600}\times100 = 6.986\%$

11 A prestressed concrete beam of rectangular section 250mm wide and 350mm deep is provided with 12 high tensioned wires of 6mm diameter, located at 10mm from the bottom of the beam and 4 similar 6mm wires at the top located at 40mm from the top of the beam. The wires are initially stretched to a stress of 900N/mm². Determine the percentage of loss of stress in the steel wires due to elastic shortening of concrete. Take $E_s = 2.1\times10^{5}N/mm^2$ and $E_c = 3.5\times10^{4}N/mm^2$.



Modular ratio $m = \frac{E_s}{E_c} = \frac{2.1\times10^5}{3.5\times10^4} = 6$

Area $= A = 350\times250 = 87500\ mm^2$

$I = \frac{bd^3}{12} = \frac{250\times350^3}{12} = 8.93\times10^8\ mm^4$.

Height of C.G of steel from the bottom edge ($\bar{y}$)

$= \frac{A_1y_1 + A_2y_2}{A_1 + A_2}$.

$= \frac{12\times\frac{\pi}{4}\times(6)^2\times(350-40) + 4\times\frac{\pi}{4}\times(6)^2\times70}{12\times\frac{\pi}{4}\times6^2 + 4\times\frac{\pi}{4}\times6^2}$

$= 130mm$

Eccentricity $e = 175 - 130 = 45mm$

Total area of steel wires $= 16\times\frac{\pi}{4}$ $12\times\frac{\pi}{4}\times(6)^2 + 4\times\frac{\pi}{4}\times(6)^2$.

$= 452.389\ mm^2$

Initial prestressing force P = stress × area.

$= 900\times452.389$

$= 407150.4\ N$

$= 407.15\ kN$.

Stress in concrete at the level of the top fibres $= \frac{P}{A} - \frac{Pey}{I}$.

$= \frac{P}{A} - \frac{P\times e\times y_t}{I}$

$= \frac{407.15\times10^3}{87500} - \frac{407150.4\times45\times180}{8.9323\times10^8}$.

$= 0.961\ N/mm^2$.

Stress in concrete at the level of the bottom fibre

$= \frac{P}{A} + \frac{P\times e\times y_b}{I}$.

$= \frac{407150.4}{87500} + \frac{407150.4\times45\times60}{8.9323\times10^8}$

$= 5.88\ N/mm^2$

Loss of stress in the top wires = $m \times f_c$

$= 6 \times 0.96$

$= 5.76\ N/mm^2$

Loss of stress in the bottom wires = $m \times f_c$

$= 6 \times 5.88$

$= 35.28\ N/mm^2$

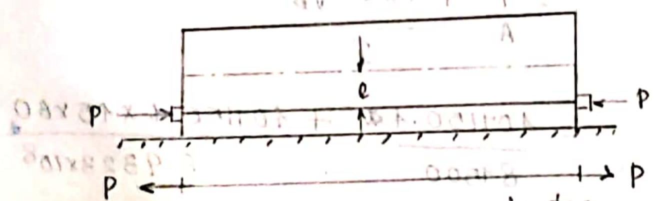
% loss of stress in the top wires = $\frac{5.76 \times 100}{900}$

$= 0.64\%$

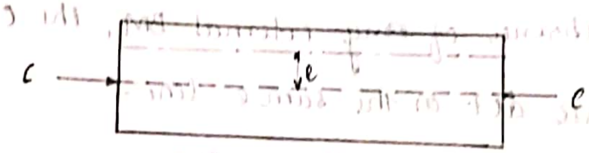
% loss of stress in the bottom wires = $\frac{35.28 \times 100}{900}$

$= 3.92\%$

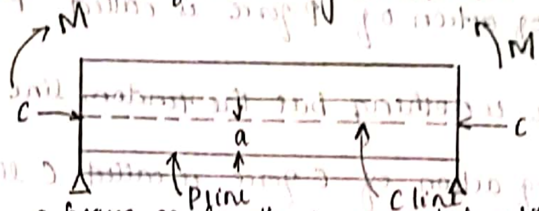
The Pressure line :



P-force acting on tendon.



C-force acting on concrete.



- Consider a beam of length L provided with a tendon at an eccentricity e.
- Suppose the beam is lying on the ground

  i.e, the beam is not subjected to any external load.
- Hence, there is no external BM on the beam.
- We should recognize the existence of the following forces which are equal

i) The P force which is in the tendon (tension)

ii) The C force which is the compressive force acting on concrete.

- Stress in concrete are produced entirely due to the C force

In the absence of any external BM, the C force and P force act on the same line.

- The line of action of P force is called P line.
- The P line is nothing but the tendon line itself.
- The line of action of C force is called C line or pressure line.
- Hence, in the absence any external BM, the P line and the C line coincide.
- Suppose the beam is subjected to a moment M, then the C line will be shifted from the P line by a distance a called the lever arm.
- a = shift of the C line from the P line.

$$= \frac{\text{external moment}}{P} = \frac{M}{P}.$$

- In other words, the effect of the moment may be considered by shifting the C line by the distance $\frac{M}{P}$.
- Now corresponding to the new position of the C line and its eccentricity the stress distribution for concrete can be determined.

Extreme stresses in concrete $= \frac{C}{A} \pm \frac{C \times \text{eccentricity of } C}{Z}$

12 A prestressed concrete beam of rectangular section is 125mm wide and 300mm deep. The beam is prestressed with a cable provided along the longitudinal centroidal axis. The effective prestressing force is 180kN. The beam carries a uniformly distributed load of 2.25kN/m including the weight of beam. Locate the pressure line for the beam. The beam has a span of 8m.

Solution.

In this case, the P line is the longitudinal centroidal axis

Reaction at each support $= \frac{wl}{2} = \frac{2.25 \times 8}{2} = \underline{9kN}$

BM at any section distant x meters from the support

$$M = 9x - \frac{2.25x^2}{2}.$$

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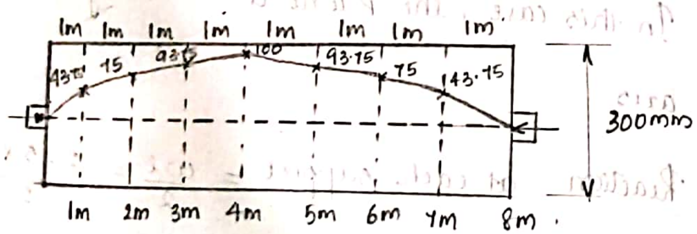
$$= 9x - 1.125x^2$$

Shift of the C line from the P line at any section distant x m from the support $= \frac{M}{P}$

$$= \frac{9x - 1.125x^2}{180}$$

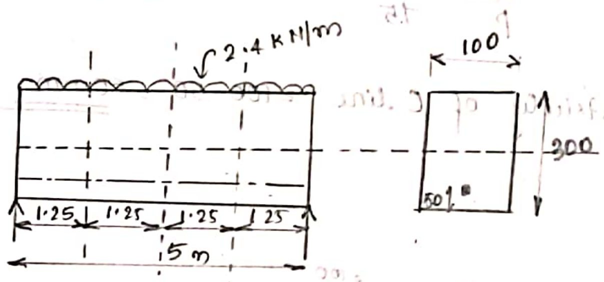
Shift of the C line from the P line for various values of x are shown in the table.

Distance from support x (meters)	Shift of C-line from P-line $\frac{9x - 1.125x^2}{180}$
0	0
1	0.04375m ≃ 43.75mm
2	0.075m = 75mm
3	0.09375m = 93.75mm
4	0.1 m = 100 mm.



13 A prestressed concrete beam 100mm x 300mm in section is prestressed with a straight cable at an eccentricity of 50mm. The effective prestress force is 75 kN.

The span of the beam is 5m and the total load on beam including the selfweight is 2.4 kN/m. Determine the pressure line at quarter span and mid span section.



Reaction at each support $= \frac{wl}{2} = \frac{2.4 \times 5}{2} = \underline{\underline{6\,kN.}}$

BM at quarter span section $(x = 1.25)$

$$BM = 6 \times x - \frac{2.4x^2}{2}$$

$$(BM)_{x=1.25} = 6 \times 1.25 - \frac{2.4 \times 1.25^2}{2} = \underline{\underline{5.625\,kNm}}$$

BM at mid span section $= 6 \times 2.5 - \frac{2.4 \times 2.5^2}{2}$

$$= \underline{\underline{7.5\,kNm}}$$

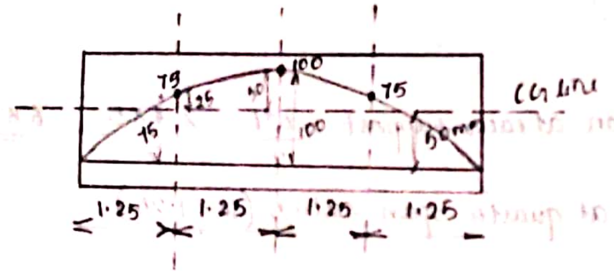
Shift of C line from P line at quarter span section $= \frac{M}{P} = \frac{5.625}{75} = 0.075m = 75mm$

$\therefore$ Eccentricity of the c line $= 75 - 50 = 25$ mm (c line from center line)

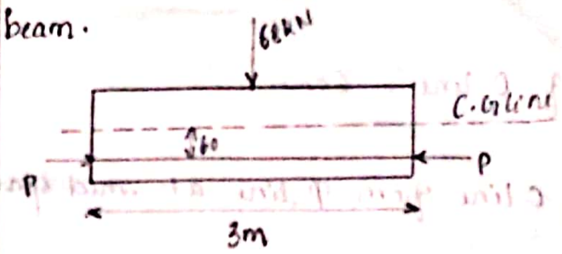
Shift of the c line from P line at mid span section

$= \frac{M}{P} = \frac{7.5}{75} = 0.1m = 100mm$

$\therefore$ Eccentricity of C line $= 100 - 50 = 50mm$ (C line from the center line) (C line from bottom)



14. A rectangular concrete beam 250mm x 300mm is prestressed by a force of 540kN at a constant eccentricity of 60mm. The beam supports a concentrated load of 68kN at the centre of span 3m. Determine the location of the pressure line at the center, quarter span and support section of the beam. Neglect the self weight of the beam.



Reaction at each support $= \frac{w}{2} = \frac{68}{2} = 34kN$

BM at support $(x = 0)$ — BM $= 34x - 68(x - 1.5)$

BM $= 34 \times 0 = 0$ kNm.

BM at quarter span section $(x = 0.75)$.

BM $= 34 \times 0.75 = 25.5$ kNm.

BM at mid span section $(x = 1.5m)$.

BM $= 34 \times 1.5 = 51$ kNm

Shift of C line from P line at quarter span

Section $= \frac{M}{P} = \frac{25.5}{68540} = 0.316 = 315mm$

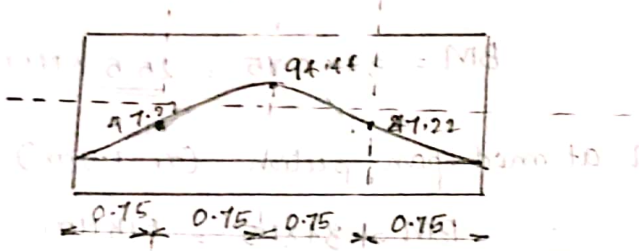
$= 47.22mm$.

Eccentricity of c line = 60mm.

Shift of the C line from P line at mid span section $= \frac{M}{P} = \frac{51}{540} = 94.44$ mm

$\therefore$ Eccentricity of C line $= 94.44 - 60$

$= 34.44$ mm



Types of prestressed concrete.

Type 1 prestressed concrete

- It is the prestressed concrete in which no tensile stresses are allowed.

Type 2 prestressed concrete.

- It is the prestressed concrete in which tensile stresses are allowed but no visible cracking is permitted.

Type 3 prestressed concrete.

- It is the prestressed concrete in which cracking is allowed but it should not affect the appearance or durability of structure.
- The acceptable limit of cracking would vary with the type of structure and environment and would vary b/w wide limits.
- For such members, as a rough guide, the surface width of cracks shouldnot, in general exceed 0.1mm

for members exposed to a particularly agressive environment and not exceed 0.2mm for all other members. Page 37

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